Symétries et stabilité de noyaux atomiques : exemples des projets

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Pathways for Future Facilities and Astrophysics ...

Symmetries and Nuclear Stability: Examples of Projects

Jerzy DUDEK, University of Strasbourg, France
Part I

General Features of the Nucleon-Nucleon Interactions
Let $\hat{x} \overset{df}{=} \{\hat{r}, \hat{p}, \hat{s}, \hat{t}\}$. Nucleon-Nucleon interactions have the form:

$$\hat{V}(\hat{x}_1, \hat{x}_2) \equiv \hat{V}_C(\hat{x}_1, \hat{x}_2) + \hat{V}_T(\hat{x}_1, \hat{x}_2) + \hat{V}_{LS}(\hat{x}_1, \hat{x}_2) + \hat{V}_{LL}(\hat{x}_1, \hat{x}_2)$$

where: $C$-central, $T$-tensor, $LS$-spin-orbit and $LL^2$-quadratic LS
Fundamental Properties of Nucleon-Nucleon Forces [1]

Let $\hat{x} \overset{df}{=} \{\hat{r}, \hat{p}, \hat{s}, \hat{t}\}$. Nucleon-Nucleon interactions have the form:

$$\tilde{V}(\hat{x}_1, \hat{x}_2) \equiv \tilde{V}_C(\hat{x}_1, \hat{x}_2) + \tilde{V}_T(\hat{x}_1, \hat{x}_2) + \tilde{V}_{LS}(\hat{x}_1, \hat{x}_2) + \tilde{V}_{LL}(\hat{x}_1, \hat{x}_2)$$

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Central Interaction ($r_{12} \equiv |\vec{r}_1 - \vec{r}_2|$)

$$\tilde{V}_C(\hat{x}_1, \hat{x}_2) = V_0(r_{12}) + V_s(r_{12}) [\vec{s}^{(1)} \cdot \vec{s}^{(2)}]$$
$$+ V_t(r_{12}) [\vec{t}^{(1)} \cdot \vec{t}^{(2)}]$$
$$+ V_{s-t}(r_{12}) [\vec{s}^{(1)} \cdot \vec{s}^{(2)}] [\vec{t}^{(1)} \cdot \vec{t}^{(2)}]$$

Invariant under rotations, translations, inversion and time-reversal
Fundamental Properties of Nucleon-Nucleon Forces [2]

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**Tensor Interaction [Non-Central]**

\[
\mathcal{S}^{(12)} \overset{df}{=} \frac{3 (\mathbf{s}_1 \cdot \mathbf{r}_{12})(\mathbf{s}_2 \cdot \mathbf{r}_{12}) - (\mathbf{s}_1 \cdot \mathbf{s}_2) \mathbf{r}_{12}^2}{r_{12}^2}
\]

and

\[
r_{12} \overset{df}{=} |\mathbf{r}_1 - \mathbf{r}_2|
\]

\[
\hat{V}_T(\hat{x}_1, \hat{x}_2) = [V_{t_0}(r_{12}) + V_{t_1}(r_{12}) \mathbf{t}_1 \cdot \mathbf{t}_2] \mathcal{S}^{(12)}
\]

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Spin-Orbit Interaction [Non-Local]

\[
\mathbf{L}^{df} \equiv \frac{1}{2}(\mathbf{r}_1 - \mathbf{r}_2) \wedge (\mathbf{p}_1 - \mathbf{p}_2), \quad r_{12}^{df} \equiv |\mathbf{r}_1 - \mathbf{r}_2| \quad \text{and} \quad \mathbf{S}^{df} \equiv \mathbf{s}_1 + \mathbf{s}_2
\]

\[
\hat{V}_{LS}(\mathbf{x}_1, \mathbf{x}_2) = [V_{LS}^{t_0}(r_{12}) + V_{LS}^{t_1}(r_{12}) \mathbf{t}_1 \cdot \mathbf{t}_2 ]\mathbf{L} \cdot \mathbf{S}
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**Quadratic Spin-Orbit Interaction [Non-Local]**

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$$\hat{V}_{LL}(\hat{x}_1, \hat{x}_2) = V_{LL}(r_{12}) \{ (\vec{s}_1 \cdot \vec{s}_2) \vec{L}^2 - \frac{1}{2} [(\vec{s}_1 \cdot \vec{L})(\vec{s}_2 \cdot \vec{L}) + (\vec{s}_2 \cdot \vec{L})(\vec{s}_1 \cdot \vec{L})] \}$$

Invariant under rotations, translations, inversion and time-reversal
Consider the motion of a system of $N = 100$ nucleons.
Dynamics: What Is Doable and What Is Not?

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- What is the expected complexity of the description?
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\[
\hat{H}(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N) \psi = E \psi
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\( 100 \times 12 = 1200 \) operators

Conclusion: It is out of question to attack by brutal force...
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100\(\times\)12 = 1200 operators

**Conclusion:**
It is out of question to attack by brutal force...
But there exist helpful mechanisms:

Symmetries and Spontaneous Symmetry Breaking
Symmetry: Exact, Approximate, Spontaneously Broken

From preceding discussion we assume that the N-N interaction

\[ \hat{V}(\hat{x}_1, \hat{x}_2) \equiv \hat{V}_C(\hat{x}_1, \hat{x}_2) + \hat{V}_T(\hat{x}_1, \hat{x}_2) + \hat{V}_{LS}(\hat{x}_1, \hat{x}_2) + \hat{V}_{LL^2}(\hat{x}_1, \hat{x}_2) \]

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Spherical Symmetry?

The Nuclear Mean Field Theory ... 
... is usually very successful. It is based on
\[ \hat{V}_{mf}(\hat{x}) = \int \psi^*(x') \hat{V}(\hat{x}, \hat{x}') \psi(x') \, dx' \]

Some or all of the above symmetries will be broken by the mean-field Hamiltonian.
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Among nearly 3000 systems known experimentally, about two hundreds are stable; they are marked in black.
In Majority of Them Spherical Symmetry is Broken

Among nearly 3000 systems known experimentally, more than 80% are strongly deformed.
A Few Important Conclusions:

- Experiments suggest that the nuclear mean-field, in general deformed, should be a dominating feature of the systems*)

*) Provided, the theory knows how to manage the corresponding Hamiltonian.

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- The mean-field is by construction a one-body operator what implies significant simplifications

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\hat{H}_{\text{nature}}(\hat{x}_1, \hat{x}_2, \ldots \hat{x}_N) \approx \hat{H}_{\text{mean field}} = \sum_{i=1}^{N} \hat{h}(\hat{x}_i)
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- From now on, effective theories can be constructed:

\[
\hat{H}_{\text{nature}} \approx \hat{H}_{\text{mean field}} + \hat{H}_{\text{residual}}
\]

*) Provided, the theory knows how to manage the corresponding Hamiltonian.
A Possible General Structure of Hamiltonians

- The unknown ‘true’ Hamiltonian is replaced by two effective ones

\[ \hat{H}_{\text{nature}} \rightarrow \hat{H} \approx \sum_{i=1}^{N} \hat{h}(\hat{x}_i) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \hat{V}^{\text{res}}(\hat{x}_i \leftrightarrow \hat{x}_j) \]

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- The form of the effective residual interactions, is influenced by microscopic theories (typically: scalar, inversion-invariant, time-even)
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\[ \hat{H}_\text{res} = \hat{V}_\text{pairing} + \hat{V}_\text{long range} + \hat{V}_\text{vib.coupling} + \ldots \]
Part II

Nuclear Stability, Spectra, Symmetries and Groups
Consider a typical outcome of the Mean-Field calculation: the shell structures and the total energies.

- Presence of sufficiently strong gaps correlates with local minima of the total nuclear energy.

The ‘Deformation Parameter’ axis represents several deformations of the mean field, e.g., \( Q_{\lambda \mu} \), \( \alpha_{\lambda \mu} \).
Reminder: Global Stability vs. Gaps in SP Spectra [1]

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Posing the problem:

Construction of our New Theory of Nuclear Stability will be equivalent to constructing a systematic method of looking for big Mean-Field Gaps

Strategical Lines:

1. Suppose Mean-Field parameters are fixed already for instance through fits to levels in spherical nuclei

2. We expect that the mean-field calculations will give bigger gaps at shapes with certain symmetries and smaller at the others

This talk is about ‘How this is going to happen’?
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Logical Consistency:

The New Theory of Nuclear Stability must contain, as particular cases, all the mechanisms of stability known so far [e.g. spherical symmetry that is SO$_3$-group]

Mathematical Implications:

All geometrical 3D-symmetries are contained in SO$_3$ as sub-groups

As a consequence: It will be sufficient to analyse all physically meaningful subgroups [point-groups] contained in SO$_3$

Comment about Practical Issues:

This is good news because all such groups are already well known ... what does not imply that we have no work to do!
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Physically Meaningful Point-Groups Contained in $SO_3$

Dashed lines indicate that the subgroup marked is not invariant.

In nuclear structure physics the point-groups used so far, mainly implicitly, are $D_2$ and $D_{2h}$ ['triaxial nuclei']. No other discrete subgroups of $SO_3$ have been explicitly used in the past.

The diagram shows possible candidate point-groups: How to profit from this information?

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Single-Particle Gaps and Groups of Symmetry
**Symmetries, Representations and Degeneracies**

- Given Hamiltonian $H$ and a group: $\mathcal{G} = \{O_1, O_2, \ldots, O_f\}$
- Assume that $\mathcal{G}$ is a symmetry group of $H$ i.e.
  \[ [H, O_k] = 0 \quad \text{with} \quad k = 1, 2, \ldots, f \]

- Let irreducible representations of $\mathcal{G}$ be $\{R_1, R_2, \ldots, R_r\}$
- Let their dimensions be $\{d_1, d_2, \ldots, d_r\}$, respectively
- Then the eigenvalues $\{\varepsilon_\nu\}$ of the problem
  \[ H \psi_\nu = \varepsilon_\nu \psi_\nu \]

appear in multiplets $d_1$-fold, $d_2$-fold ... degenerate
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Outline of our New Theory of Nuclear Stability

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Single-Particle Structure and Global Stability of Nuclei

Single-Particle Gaps and Groups of Symmetry

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Roughly: The average level spacings within an irrep increase by a factor of 6. The total spectrum may present big unprecedented gaps.
Symmetries and Gaps in Nuclear Context: Schematic

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## Symmetries, Representations and Degeneracies

<table>
<thead>
<tr>
<th>No.</th>
<th>Group $G_T^n$</th>
<th>No. Irr.</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>01.</td>
<td>$O^{D}_{h}$</td>
<td>6</td>
<td>4 x 2D and 2 x 4D</td>
</tr>
<tr>
<td>02.</td>
<td>$O^{D}$</td>
<td>3</td>
<td>2 x 2D and 1 x 4D</td>
</tr>
<tr>
<td>03.</td>
<td>$T^{D}_{d}$</td>
<td>3</td>
<td>2 x 2D and 1 x 4D</td>
</tr>
<tr>
<td>04.</td>
<td>$C^{D}_{6h}$</td>
<td>12 → 6</td>
<td>12 x 1D</td>
</tr>
<tr>
<td>05.</td>
<td>$D^{D}_{6h}$</td>
<td>6</td>
<td>6 x 2D</td>
</tr>
<tr>
<td>06.</td>
<td>$T^{D}_{h}$</td>
<td>6</td>
<td>6 x 2D</td>
</tr>
<tr>
<td>07.</td>
<td>$D^{D}_{4h}$</td>
<td>4</td>
<td>4 x 2D</td>
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<tr>
<td></td>
<td>$D^{D}_{2h}$</td>
<td>2</td>
<td>2 x 2D (reference)</td>
</tr>
</tbody>
</table>

Table: *Point-groups and their Irreducible Representations [Part 1]*.
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<td>$T^D$</td>
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</tr>
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Among the ‘standard 32 point-groups’, subgroups of $SO_3$, there are 16 (!) that satisfy more favourably the big-gap criteria than the ‘reference’ group $D_{2h}$ ['usual’ tri-axial nuclei]

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Nuclear $D_{2d}$-Group: 3D Examples

The nuclear $D_{2d}$-symmetric shapes have been predicted to coexist with the axial super-deformed shapes at high spins (JD and X. Li)

Observations:
- Nuclear elongation in the range of $\alpha_{20} \sim (0.45 \rightarrow 0.55)$;
- Barriers between the coexisting minima $\sim (1 \rightarrow 2) \text{ MeV}$
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The nuclear $D_{3d}$-symmetric shapes are expected at high spins; they correspond to superposition of $\alpha_{20}$ and $\alpha_{43}$ (inversion symmetric).

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- Probably seen already (remain mis-interpreted as tri-axiality)
Nuclear $C_{3h}$-Group ('Octupole'): 3D Examples

The nuclear $C_{3h}$-symmetric shapes are expected at high spins; they correspond to superposition of $\alpha_{20}$ and $\alpha_{33}$

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- Probably seen already (and mis-interpreted in terms of $I^\pi = 3^-$)

Figure: Elongation axis
Figure: Perspective 1
Figure: Perspective 2
Part III

What Do We Need at This Stage?

Physical Realisation of the Mathematical Guide-lines:
- Learn constructing the Mean-Field Hamiltonians invariant under pre-selected point-group → examine new quantum [shell] effects
- Formulate the theoretical predictions [identification criteria]

Verification by Experiments:
- Look for the experimental evidence in agreement with the criteria
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- Given group $G = \{\hat{O}_1, \hat{O}_2, \ldots \hat{O}_f\}$. How to construct a realistic Hamiltonian invariant under all transformations in $G$?

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- The condition of invariance:

$$\Sigma \xrightarrow{\hat{O}} \Sigma' \equiv \Sigma \quad \forall \quad \hat{O}$$

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New Theory: Verification, Relation to Experiment

Specific Point-Group Symmetry Realisations (Order $\lambda$)

**Invariant Mean-Field Hamiltonian [2]**

- In what follows we will need a representation of the operators $\hat{O} \in G$ adapted to the action on spherical harmonics $Y_{\lambda\mu}(\vartheta, \varphi)$.

- The action of proper rotations can be written down as

  $$\hat{O} \rightarrow R(\Omega) \equiv \exp(i\alpha\hat{j}_z + i\beta\hat{j}_y + i\gamma\hat{j}_z')$$

- Using this notation the invariance condition takes the form

  $$\sum_{\lambda=2}^{\lambda_{\text{max}}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} [\hat{O} Y_{\lambda\mu}(\vartheta, \varphi)] = \sum_{\lambda=2}^{\lambda_{\text{max}}} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} \sum_{\mu'=-\lambda}^{\lambda} D_{\mu'\mu}^{\lambda}(\Omega) Y_{\lambda\mu'}(\vartheta, \varphi)$$

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A Basis for Tetrahedral Symmetry: Phenomenological

Only special combinations of spherical harmonics may form a basis for surfaces with tetrahedral symmetry and only odd-order:

Three Lowest Order Solutions: 

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A Basis for Tetrahedral Symmetry: Constrained HF

Only special combinations of multipole moments may form a basis for nuclei with tetrahedral symmetry and only odd-order:

Three Lowest Order Solutions:

\[
\begin{align*}
\lambda &= 3 : & Q_3, \pm 2 & \equiv Q_3 \\
\lambda &= 5 : & \text{no solution possible} \\
\lambda &= 7 : & Q_7, \pm 2 & \equiv Q_7; & Q_7, \pm 6 & \equiv -\sqrt{\frac{11}{13}} \cdot Q_7 \\
\lambda &= 9 : & Q_9, \pm 2 & \equiv Q_9; & Q_9, \pm 6 & \equiv +\sqrt{\frac{28}{198}} \cdot Q_9
\end{align*}
\]
Let us recall one of the magic forms introduced long time by Plato. The implied symmetry leads to the tetrahedral group denoted $T_d$.

A tetrahedron has four equal walls. Its shape is invariant with respect to 24 symmetry elements. Tetrahedron is not invariant with respect to the inversion. Of course nuclei cannot be represented by a sharp-edge pyramid... but rather in a form of a regular spherical harmonic expansion:

$$\mathcal{R}(\vartheta, \varphi) = R_0 \left\{ 1 + \alpha_{3+2} (Y_{3+2} + Y_{3-2}) + \alpha_{72} \left[ (Y_{7+2} + Y_{7-2}) - \sqrt{\frac{11}{13}} (Y_{7+6} + Y_{7-6}) \right] \right\}$$

one parameter 3rd order

one parameter 7th order
A Basis for Octahedral Symmetry: Phenomenological

Only special combinations of spherical harmonics may form a basis for surfaces with octahedral symmetry and only in even-orders:

Three Lowest Order Solutions:

- \( \lambda = 4 \): \( \alpha_{40} \equiv o_4; \quad \alpha_{4,\pm4} \equiv \pm \sqrt{\frac{5}{14}} \cdot o_4 \)
- \( \lambda = 6 \): \( \alpha_{60} \equiv o_6; \quad \alpha_{6,\pm4} \equiv -\sqrt{\frac{7}{2}} \cdot o_6 \)
- \( \lambda = 8 \): \( \alpha_{80} \equiv o_8; \quad \alpha_{8,\pm4} \equiv \sqrt{\frac{28}{198}} \cdot o_8; \quad \alpha_{8,\pm8} \equiv \sqrt{\frac{65}{198}} \cdot o_8 \)
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- For $\lambda = 8$:
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Jerzy DUDEK, University of Strasbourg, France

Symmetries and Nuclear Stability: Examples of Projects
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Introducing Nuclear Octahedral Symmetry

Let us recall one of the magic forms introduced long time by Plato. The implied symmetry leads to the *octahedral group* denoted $O_h$.

An octahedron has 8 equal walls. Its shape is invariant with respect to 48 symmetry elements that include inversion. However, the nuclear surface cannot be represented in the form of a diamond $→→→→→→→→$ but rather in a form of a regular spherical harmonic expansion:

$$R(\theta, \varphi) = R_0 \left\{ 11 + \alpha_{40} \left[ Y_{40} + \sqrt{\frac{5}{14}} (Y_{4+4} + Y_{4-4}) \right] + \alpha_{60} \left[ Y_{60} - \sqrt{\frac{7}{2}} (Y_{6+4} + Y_{6-4}) \right] \right\}$$

one parameter 4th order

$→→→→→→→→$

one parameter 6th order

*Jerzy DUDEK, University of Strasbourg, France*
New Theory: Verification, Relation to Experiment

Specific Point-Group Symmetry Realisations (Order $\lambda$)

**Example: Octahedral Symmetry - Proton Spectra**

Double group $O^D_h$ has four 2-dimensional and two 4-dimensional irreducible representations $\rightarrow$ six distinct families of levels

**Figure:** Full lines correspond to 4-dimensional irreducible representations - they are marked with double Nilsson labels. Observe huge gap at $Z=70$. 

Jerzy DUDEK, University of Strasbourg, France

Symmetries and Nuclear Stability: Examples of Projects
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Figure: Full lines correspond to 4-dimensional irreducible representations - they are marked with double Nilsson labels. Observe huge gap at $N=114$. 

Jerzy DUDEK, University of Strasbourg, France

Symmetries and Nuclear Stability: Examples of Projects
**Example: Results with the HFB Solutions in RE**

The HFB results for tetrahedral solutions in light Rare-Earth nuclei

\[ \alpha_{4,0} \equiv o_4, \quad \alpha_{4,\pm 4} \equiv -\sqrt{\frac{5}{14}} o_4 \]

<table>
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<th>N</th>
<th>$\Delta E$ (MeV)</th>
<th>$Q_{32}$ ($b^{3/2}$)</th>
<th>$Q_{40}$ ($b^2$)</th>
<th>$Q_{44}$ ($b^2$)</th>
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<tr>
<td>64</td>
<td>86</td>
<td>-1.387</td>
<td>0.941817</td>
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Symmetries and Nuclear Stability: Examples of Projects
Example: Results with the HFB Solutions in Actinide

The HFB results for tetrahedral solutions in the Actinide nuclei

\[ \alpha_{4,0} \equiv o_4, \quad \alpha_{4,\pm4} \equiv -\sqrt{5/14} o_4 \]

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Table: Octahedral deformations of the second order compatible with tetrahedral deformation in \(^{226}\)Th with two Skyrme parameterisations.

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Group Theory and Geometry for Historians

- The symbol of 'beauty in symmetry' are five Platonic Figures
- There exist only five regular convex (=platonic) polyhedra: tetrahedron, cube, octahedron, icosahedron & dodecahedron
- As it seems, neolithic people from Scotland have developed the five Platonic solids about 1000-3000 years before Plato (stone models in Ashmolean Museum, Oxford) ... in religious context
- In what follows we stick to the aspect of beauty and nuclear reality - similarity to any other context will be purely accidental
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Part IV

Predictions and the Experimental Verification
Multipole Moments as Functionals of the Density

- Nuclear surface $\Sigma$ is defined in terms of multipole deformations:

$$\Sigma : \quad R(\vartheta, \varphi) = R_0 \left[1 + \sum \lambda \sum \mu \alpha_{\lambda\mu} Y_{\lambda\mu}(\vartheta, \varphi)\right]$$

- Given uniform density $\rho_{\Sigma}(\vec{r})$ defined using the surface $\Sigma$

$$\rho_{\Sigma}(\vec{r}) = \begin{cases} 
\rho_0 & : \quad \vec{r} \in \Sigma \\
0 & : \quad \vec{r} \notin \Sigma 
\end{cases}$$

- Express the multipole moments as usual by

$$Q_{\lambda\mu} = \int \rho_{\Sigma}(\vec{r}) r^\lambda Y_{\lambda\mu} \, d^3\vec{r}$$

- We will calculate the quadrupole moments as functions of $\alpha_{3\mu}$
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Multipole Moments as Functionals of the Density

For small deformations we use Taylor expansion:

\[ Q_{\lambda\mu}(\alpha) \approx Q_{\lambda\mu}\bigg|_{\alpha=0} + Q_{\lambda\mu}'\bigg|_{\alpha=0} \Delta \alpha + \frac{1}{2} Q_{\lambda\mu}''\bigg|_{\alpha=0} \Delta \alpha \Delta \alpha \]

We set \( \lambda = 2, \mu = 0 \) and \( \lambda_1 = \lambda_2 = 3 \) and obtain

\[ \alpha_{30} : \quad Q_{20} = \frac{15}{2\sqrt{5\pi}} \cdot \alpha_{30}^2 \cdot \rho_0 R_0^5 \]
\[ \alpha_{31} : \quad Q_{20} = \frac{15}{4\sqrt{5\pi}} \cdot \alpha_{3+1} \alpha_{3-1} \cdot \rho_0 R_0^5 \]
\[ \alpha_{32} : \quad Q_{20} = 0 \]
\[ \alpha_{33} : \quad Q_{20} = \frac{125}{12\sqrt{5\pi}} \cdot \alpha_{3+3} \alpha_{3-3} \cdot \rho_0 R_0^5 \]

Conclusion: Among \( \lambda = 3 \) def. only \( \alpha_{32} \) leads to \( Q_2 \equiv 0 \) !!!
Multipole Moments as Functionals of the Density

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\[
Q_{\lambda\mu}(\alpha) \approx Q_{\lambda\mu}\bigg|_{\alpha=0} + Q'_{\lambda\mu}\bigg|_{\alpha=0} \Delta\alpha + \frac{1}{2} Q''_{\lambda\mu}\bigg|_{\alpha=0} \Delta\alpha \Delta\alpha
\]

- We set \(\lambda = 2\), \(\mu = 0\) and \(\lambda_1 = \lambda_2 = 3\) and obtain

\[
\begin{align*}
\alpha_{30} : & \quad Q_{20} = 15/(2\sqrt{5\pi}) \cdot \alpha_{30}^2 \cdot \rho_0 R_0^5 \\
\alpha_{31} : & \quad Q_{20} = 15/(4\sqrt{5\pi}) \cdot \alpha_{3+1} \alpha_{3-1} \cdot \rho_0 R_0^5 \\
\alpha_{32} : & \quad Q_{20} = 0 \\
\alpha_{33} : & \quad Q_{20} = 125/(12\sqrt{5\pi}) \cdot \alpha_{3+3} \alpha_{3-3} \cdot \rho_0 R_0^5
\end{align*}
\]

- **Conclusion:** Among \(\lambda = 3\) def. only \(\alpha_{32}\) leads to \(Q_2 \equiv 0\) !!!
Rotational Bands without Electric E2 Transitions
Experiments and TetraNuc Collaboration

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\alpha_{31} : & \quad Q_{20} = \frac{15}{4\sqrt{5\pi}} \cdot \alpha_{3+1} \alpha_{3-1} \cdot \rho_0 R_0^5 \\
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\alpha_{33} : & \quad Q_{20} = \frac{125}{12\sqrt{5\pi}} \cdot \alpha_{3+3} \alpha_{3-3} \cdot \rho_0 R_0^5
\end{align*}
\]

- **Conclusion:** Among $\lambda = 3$ defs. only $\alpha_{32}$ leads to $Q_2 \equiv 0$ !!!
Similar Results with the Microscopic Hamiltonian

Microscopic Multipole Moments: \[ Q_{20}(\alpha_{3\mu}) = \int \psi^*(\tau) \hat{Q}_{20} \psi(\tau) d\tau \]

Observe that \( Q_{20}(\alpha_{32}) \) vanishes identically in the micro case as well.
Octupole Shapes and Point-Groups

• Each of 4 octupole parameters $\alpha_{3\mu}$ defines a family of surfaces
• Each surface has certain symmetry (i.e. invariance) properties
• Those symmetry properties define the corresponding groups:

<table>
<thead>
<tr>
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<tr>
<td>$\alpha_{30}$</td>
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</tr>
<tr>
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Jerzy DUDEK, University of Strasbourg, France
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We Have the Smoking-Gun Signatures Almost There

Valence particles cause a certain quadrupole polarisation

Additional polarisation caused by Coriolis spin alignments

Spin-alignment will cause additional quadrupole polarisation

Jerzy DUDEK, University of Strasbourg, France
We can now formulate further experimental criteria!

The Story of the ‘Smoking Guns’

- Tetrahedral nuclei are deformed → they produce collective rotation
- The lowest order $T_d$–symmetry is $Y_{3\pm 2}$ → negative parity bands
- At the exact symmetry limit $Q_2$ moments must vanish! Therefore:
- There must exist negative-parity bands without E2 transitions !!!

We suggest looking for the collective negative parity bands without ‘rotational’ (E2) transitions. The question – Where?
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1. After inspecting many single-particle diagrams in function of tetrahedral deformation we read-out all the magic numbers.

2. The tetrahedral symmetric nuclei are predicted to be particularly stable around magic closures:

\[ \{Z_t, N_t\} = \{32, 40, 56, 64, 70, 90, 136\} \]

3. ... and more precisely around the following nuclei:

\[ ^{64}_{32} \text{Ge}_{32}, \quad ^{72}_{32} \text{Ge}_{40}, \quad ^{88}_{32} \text{Ge}_{56}, \quad ^{80}_{40} \text{Zr}_{40}, \quad ^{110}_{40} \text{Zr}_{70}, \quad ^{112}_{56} \text{Ba}_{56}, \]

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Tetrahedral Stability; Tetrahedral Magic Numbers

Tetrahedral Symmetry Induced Magic Numbers

- N = 40
- N = 56
- N = 70
- N = 112
- N = 136
- Z = 40
- Z = 56
- Z = 64
- Z = 70
- Z = 90

Jerzy DUDEK, University of Strasbourg, France
Tetrahedral Stability; Tetrahedral Magic Numbers

Tetrahedral Symmetry Induced Magic Numbers

- N ≤ 90
- N = 70
- N = 56
- N = 40
- Z = 90
- Z = 70
- Z = 64
- Z = 56
- Z = 40

Proton Number
Neutron Number

Jerzy DUDEK, University of Strasbourg, France
Symmetries and Nuclear Stability: Examples of Projects
Partial Decay of $^{156}$Gd and Vanishing $Q_2$-Moments

From $I^\pi = 9^-$ down - no E2-transitions are observed despite tries

<table>
<thead>
<tr>
<th>Process</th>
<th>Refs</th>
<th>Last Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{156}$EU B- DECAY</td>
<td>43</td>
<td>1995</td>
</tr>
<tr>
<td>$^{156}$TB EC DECAY</td>
<td>38</td>
<td>1995</td>
</tr>
<tr>
<td>$^{150}$ND(13C,A3NG)</td>
<td>2</td>
<td>2001</td>
</tr>
<tr>
<td>$^{154}$SM(A,2NG)</td>
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<td>2001</td>
</tr>
<tr>
<td>$^{154}$GD(T,P)</td>
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<td>1989</td>
</tr>
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<td>$^{155}$GD(N,G)</td>
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<td>2000</td>
</tr>
<tr>
<td>$^{155}$GD(D,P)</td>
<td>2</td>
<td>1994</td>
</tr>
<tr>
<td>$^{156}$GD(G,G'),(E,E')</td>
<td>27</td>
<td>2000</td>
</tr>
<tr>
<td>$^{156}$GD(MU,G)</td>
<td>1</td>
<td>1971</td>
</tr>
<tr>
<td>$^{156}$GD(N,N')</td>
<td>3</td>
<td>1996</td>
</tr>
<tr>
<td>$^{156}$GD(P,P'),(D,D')</td>
<td>5</td>
<td>1989</td>
</tr>
<tr>
<td>COULOMB EXCITATION</td>
<td>25</td>
<td>1993</td>
</tr>
<tr>
<td>$^{157}$GD(P,D),(3HE,A)</td>
<td>2</td>
<td>1984</td>
</tr>
<tr>
<td>$^{157}$GD(D,T)</td>
<td>1</td>
<td>1993</td>
</tr>
<tr>
<td>$^{158}$GD(P,T)</td>
<td>8</td>
<td>1982</td>
</tr>
</tbody>
</table>

According to C. W. Reich, *Nucl. Data Sheets* **99** 753 (2003) a few dozens among those refs have been used to deduce the level scheme on the left...
Tetrahedral/Octahedral Shapes Have No $Q_2$-Moments

At the exact tetrahedral symmetry the quadrupole moments vanish

Equilibrium shape $t_1 = 0.15$

...but, $E2$-intensities are expected to grow with spin (Coriolis polarisation)
More Negative-Parity Bands with No Q$_2$-Moments

Despite numerous tries nobody has ever succeed in observing E2’s

The bands are identified thanks to the E1 transitions to the GSBs
More Negative-Parity Bands with No $Q_2$-Moments

Despite numerous tries nobody has ever succeed in observing $E2$'s

The bands are identified thanks to the $E1$ transitions to the GSBs

Jerzy DUDEK, University of Strasbourg, France
Evidence for Vanishing E2 Transitions in Actinides

The E2 transitions not seen in \(^{230-234}\text{U}\), while seen in \(^{236}\text{U}\); the experimental conditions (\(\gamma\) and \(ec\)) are the same or comparable.
Observe the ‘tetrahedral’ band patterns (vanishing E2-transitions) in $^{230-232}\text{U}$ in both the negative and positive parities!

Jerzy DUDEK, University of Strasbourg, France
Symmetries and Nuclear Stability: Examples of Projects
According to a simplified way of thinking, when all deformations tend to zero ($\alpha, \lambda, \mu \rightarrow 0$) then $Q_2 \rightarrow 0$ and $Q_1 \rightarrow 0$ and we are confronted with an ill-defined mathematical problem

$$\lim_{\alpha \rightarrow 0} \frac{B(E2)}{B(E1)} = ??? \quad (\text{undefined symbol } 0/0) !!!$$

However, because of the residual polarisations in terms of quadrupole deformation and of induced dipole moments at the band-heads we have

$$\lim_{\alpha \rightarrow 0} \frac{B(E2)}{B(E1)} = \frac{B_{\text{res}}(E2)}{B_{\text{res}}(E1)} \equiv B_0 \neq 0$$
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We Have the Smoking-Gun Signatures Almost There ...

In other words, we expect a spin dependence: \( \frac{B(E2)}{B(E1)} \sim B_0 + B_1 \cdot I \)

**Conclusion:** Tetrahedral symmetry must always be accompanied by static or dynamic quadrupole deformations at \( \alpha_{20} \neq 0 \) and \( \alpha_{22} \neq 0 \)
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<td>-</td>
<td>50</td>
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</tr>
<tr>
<td>17(^-)</td>
<td>-</td>
<td>16</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>15(^-)</td>
<td>-</td>
<td>6</td>
<td>-</td>
<td>60</td>
<td>24</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>13(^-)</td>
<td>14</td>
<td>7</td>
<td>15</td>
<td>18</td>
<td>23</td>
<td>-</td>
<td>17</td>
</tr>
<tr>
<td>11(^-)</td>
<td>4</td>
<td>15</td>
<td>5</td>
<td>9</td>
<td>0</td>
<td>10</td>
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</tr>
<tr>
<td>9(^-)</td>
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Possible El-Magnetic Signs of Tetrahedral Symmetry

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<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>17$^-$</td>
<td>-</td>
<td>16</td>
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<td>+0.4</td>
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<td>15$^-$</td>
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<td>60</td>
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### Conclusion: Tetrahedral bands in Rare Earth nuclei behave very differently as compared e.g. to 'classical' octupole $^{222}$Th band!
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<td>18</td>
<td>23</td>
<td>0</td>
<td>0</td>
<td>+0.3</td>
</tr>
<tr>
<td>11$^-$</td>
<td>4</td>
<td>15</td>
<td>5</td>
<td>9</td>
<td>0</td>
<td>10</td>
<td>11</td>
<td>+0.4</td>
</tr>
<tr>
<td>9$^-$</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>11</td>
<td>10</td>
<td>+0.4</td>
</tr>
<tr>
<td>7$^-$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+0.4</td>
</tr>
</tbody>
</table>

Conclusion: Tetrahedral bands in Rare Earth nuclei behave very differently as compared e.g. to 'classical' octupole $^{222}$Th band!

Jerzy DUDEK, University of Strasbourg, France
**Example: This Is Not Our Effect [Pear-Shape]**

Deformation\(\alpha_{20}\)

\[
E(fyu) + \text{Shell[e]} + \text{Correlation}[PNP]
\]

\[
\begin{array}{c}
E\text{[MeV]} \\
\end{array}
\]

\[
\begin{array}{c}
\text{Emin} = -7.58, \text{Eo} = -3.07
\end{array}
\]

\[
\begin{array}{c}
222\text{Th}_{132}
\end{array}
\]

Jerzy DUDEK, University of Strasbourg, France

Symmetries and Nuclear Stability: Examples of Projects
 THEORY PREDICTIONS AND EXPERIMENTAL VERIFICATION

EXAMPLE: THIS IS NOT OUR EFFECT [PEAR-SHAPED]

\[
E(fyu) + \text{Shell} + \text{Correlation}[\text{PNP}]
\]

\[
\frac{B(E2)_{\text{in}}}{B(E1)_{\text{out}}} \times 10^6 \text{ eFm}
\]

\begin{array}{|c|c|}
\hline
\text{Spin} & \text{\textsuperscript{222}Th} \\
\hline
19^- & +0.3 \\
17^- & +0.4 \\
15^- & +0.4 \\
13^- & +0.3 \\
11^- & +0.4 \\
09^- & +0.4 \\
07^- & +0.4 \\
\hline
\end{array}

\text{Experiments and TetraNuc Collaboration}

\text{Symmetries and Nuclear Stability: Examples of Projects}

\text{Jerzy DUDEK, University of Strasbourg, France}
Tetrahedral-Symmetry Candidates in the Actinides

Experiment: Three types of situations correspond to three colours:
- **[red]**-tetrahedral,
- **[yellow]**-octupole,
- **[green]**-both.

<table>
<thead>
<tr>
<th></th>
<th>N →</th>
<th>130</th>
<th>132</th>
<th>134</th>
<th>136</th>
<th>138</th>
<th>140</th>
<th>142</th>
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<td></td>
<td>Cm</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>244</td>
<td>246?</td>
<td>248</td>
</tr>
<tr>
<td></td>
<td>Pu</td>
<td></td>
<td></td>
<td>236</td>
<td>238</td>
<td>240</td>
<td>242?</td>
<td>244</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Th</td>
<td>220</td>
<td>222</td>
<td>224</td>
<td>226</td>
<td>228</td>
<td>230</td>
<td>232</td>
<td>234</td>
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<td>222</td>
<td>224</td>
<td>226</td>
<td>228</td>
<td>230</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Rn</td>
<td>216</td>
<td>218</td>
<td>220</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Nearly half of the experimental data on the Actinidae nuclei do not manifest the E2-transitions in the negative-parity bands!
Table: Experimental ratios $B(E2)/B(E1) \times 10^6 [fm^2]$, for intra-band $E2$ transitions vs. inter-band $E1$ transitions. Meaning of symbols: “−” - state has not been observed; “?” - intensities to calculate the branching ratios not available; “(?)” - known information insufficient to obtain error bars.

<table>
<thead>
<tr>
<th>State</th>
<th>$^{220}$Th</th>
<th>$^{222}$Th</th>
<th>$^{224}$Th</th>
<th>$^{226}$Th</th>
<th>$^{228}$Th</th>
<th>$^{152}$Gd</th>
<th>$^{156}$Gd</th>
</tr>
</thead>
<tbody>
<tr>
<td>21−</td>
<td>no E1</td>
<td>0.2(?)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>19−</td>
<td>no E1</td>
<td>0.3(?)</td>
<td>-</td>
<td>-</td>
<td>2.0(5)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17−</td>
<td>no E1</td>
<td>0.4(2)</td>
<td>0.3(1)</td>
<td>2.3(4)</td>
<td>2.0(4)</td>
<td>no E1</td>
<td>-</td>
</tr>
<tr>
<td>15−</td>
<td>0.9(2)</td>
<td>0.4(2)</td>
<td>0.4(1)</td>
<td>?</td>
<td>?</td>
<td>no E1</td>
<td>16(3)</td>
</tr>
<tr>
<td>13−</td>
<td>0.2(1)</td>
<td>0.3(2)</td>
<td>0.5(1)</td>
<td>2.0(2)</td>
<td>?</td>
<td>no E1</td>
<td>14(?)</td>
</tr>
<tr>
<td>11−</td>
<td>0.5(1)</td>
<td>0.4(2)</td>
<td>0.4(1)</td>
<td>2.0(2)</td>
<td>?</td>
<td>no E2</td>
<td>4(?)</td>
</tr>
<tr>
<td>9−</td>
<td>0.4(1)</td>
<td>0.4(2)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>no E2</td>
<td>no E2</td>
</tr>
<tr>
<td>7−</td>
<td>0.4(1)</td>
<td>0.4(3)</td>
<td>?</td>
<td>?</td>
<td>?</td>
<td>no E2</td>
<td>no E2</td>
</tr>
</tbody>
</table>
Figure: *RIKEN Superconducting Ring Cyclotron (SRC)*
Figure: *RIKEN Ring Cyclotron*
Figure: *BigRIPS-II* a medium-acceptance separator.
Figure: *Gammasphere is the world’s most powerful gamma-spectrometer (surrounded by the very friendly, warm, stimulating, American atmosphere)*
Figure: Gammasphere ‘opened’ showing the area where the beam hits the target; This detector has worked and will be working for us to possibly discover the tetrahedral symmetry in subatomic physics

Jerzy DUDEK, University of Strasbourg, France

Symmetries and Nuclear Stability: Examples of Projects
We Have Launched the Collaboration 'TetraNuc'

A few points about the TETRANUC collaboration:

Goal: Demonstrate the existence and study the consequences of tetrahedral symmetry in subatomic physics and astrophysics

LABORATORIES TODAY: IPHC Strasbourg, Ganil, ILL-Grenoble, IPN-Lyon, IPN-Orsay, CSNSM-Orsay, CEA-Saclay, IFJ PAN Cracow, GSI-Darmstadt, U of Jyvaslyla, LN Legnaro, U of Lublin, U of Mainz, ORNL and Knoxville, U of Surrey and U of Warsaw

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Most Urgent: Proposals to Study Branching Ratios

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The LOGO (!) of the new collaboration suggests our principal interest:

Find for the first time the experimental evidence of the tetrahedral symmetry in subatomic physics and/or astrophysics.

First proposals: To study the suspect tetrahedral band in $^{156}$Gd. Using known transition energies aim at as precise as possible $B(E2)/B(E1)$ ratios, also $Q_2$. 

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Part V
Summary and Conclusions
The Strongest Tetrahedral Islands Predicted by Theory

In the Actinide region, most of the so-called octupole bands have never seen their E2 transitions in experiment [detailed discussion].
The Strongest Tetrahedral Islands Predicted by Theory

In the Rare Earth Region
the Sm, Gd and Dy nuclei \([Z=62,64,66]\)
manifest negative parity bands
without E2 transitions [see details]
The Strongest Tetrahedral Islands Predicted by Theory

In the Zirconium region, several nuclei manifest the largest ever octupole transitions \([B(E3) \sim (20–60)\text{W.u.}]\).
Summary

• We presented what we call the *New Theory of Nuclear Stability* based on the nuclear mean-field concepts and group theory.

• On its basis we suggest that the nuclear stability is underlined by spatial symmetries with high number of irreducible representations.

• ... rather than multipole expansion, prolate/oblate shape coexistence etc. - although the latter are a particular case of the former.

• We have illustrated the new theory of stability with two high-rank point-groups: *octahedral* - and its *tetrahedral* sub-group.
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• We have demonstrated through realistic calculations that several point-groups so far never considered in nuclear structure physics lead to very strong shell effects

• In particular: tetrahedral symmetry minima imply the presence of negative parity bands with vanishing E2 transitions

• We have found the presence of those bands in the existing literature in full agreement with our general predictions

• It is suggested that the 'octupole effects', considered so far in the literature, separate into two categories: the ‘traditional’ ('pears') and ‘tetrahedral’ ('pyramids')
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Perspectives

• We are going to study the relatively low spin states, \( I \leq 20 \hbar \) at relatively high excitations - in Rare Earth and Actinide Nuclei.

• Of high priority are the life-time measurements of the absolute values of quadrupole and dipole moments of the tetrahedral bands.

• We have performed recently three experiments along these lines \([^{156}\text{Gd in Legnaro and Jyväskylä}, ^{156}\text{Dy at Argonne Nat. Laboratory}]\)

• We are extending the new symmetry ideas to the super-heavy nuclei \([\text{extensive calculations for nuclei around } Z\sim118 \text{ and } N\sim178]\);
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I wish to thank for the support and participation in experiments as well as help in theoretical development the following Colleagues:


A. Góźdź, A. Dobrowolski - University of Lublin, Poland

N. Dubray - CEA, Bruyères-le-Châtel, F

N. Alahari, G. de France, M. Rejmund - GANIL, Caen, F

B. Lauss - ILL, Grenoble, F

N. Redon, Ch. Schmitt, O. Stézowski, D. Q. Tuyen - IPN, Lyon, F


A. Astier, G. Georgiev - CSNSM, Orsay, F

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G. de Angelis, A. Gadea, D.R. Napoli, J.J. Valiente-Dobon, F. Della Vedova, R. Orlandi, E. Sahin - INFN, Laboratori Nazionali di Legnaro, Legnaro, Italy

D. Mengoni, F. Recchia, S. Aydin, R. Menegazzo, D. Bazzacco, E. Farnea, S. Lunardi, C. Ur - Dipartimento di Fisica and INFN, Sezione di Padova, Padova, Italy

Y. R. Shimizu - Kyushu University, Fukuoka, JP

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Last but not least many thanks to an extremely dynamic contributions from our colleagues from the USA

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N. Schunck - University of Tennessee, USA
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C.J. Chiara, F.G. Kondev - Nuclear Engineering Division, Argonne National Laboratory, Argonne, IL 60439
P.E. Garrett - Department of Physics, University of Guelph, Guelph, Ontario, Canada
W.D. Kulp - School of Physics, Georgia Institute of Technology, Atlanta, GA 30332
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M.A. Riley - Department of Physics, Florida State University, Tallahassee, FL 32306L
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and was revived some 10 years later with a series of new articles

We had a workshop about these issues in 2006 at Trento, Italy

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