Fidélité quantique pour les systèmes à N corps confinés

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• **Electron dynamics in finite-size systems**
  
  – For ex.: Semiconductor “quantum wells” and “quantum dots”
  – Nanometric devices containing one or more electrons
  – Various types of confinement: parabolic, square well, …

Display a number interesting properties:

- **Finite size** (*due to confinement*)
- **Quantum** (*size of wave function ~ size of well*)
- **Collective** (*electrons interact*)
- **Nonlinear** (*strong excitations*)
• We want to characterize the stability of a quantum system against a small perturbation, simulating its “environment”.

• **Open quantum systems (non-Hamiltonian: quantum Fokker-Planck)**
  – The evolution is *non-unitary*
  – **Decoherence**: deterioration of the “purity” of a quantum state via interaction to its environment.
  – Pure state \( t = 0 \) → Mixed state \( t > 0 \)

• **Closed quantum systems (Hamiltonian)**
  – The evolution is *unitary*
  – Quantum coherence is measured by the **quantum fidelity** (see next slide...)
Quantum fidelity

The wave function evolves according to the Schrödinger equation:

\[ H = H_0 + \delta H \]

Quantum fidelity measures the overlap of the two final states:

\[ F(t) = \left| \langle \psi_{H_0}(t) | \psi_H(t) \rangle \right|^2. \]
Quantum fidelity for one-particle systems

- Single particle in a given (classically chaotic) Hamiltonian.
- For medium-sized perturbations, the fidelity decays exponentially, with a rate equal to the classical Lyapunov exponent
- The rate is independent on the perturbation $\delta H$

Quantum fidelity for N-particle systems: model

- **Wigner-Poisson equations** (single-band, effective-mass approximation)

\[
\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{i m_*}{2 \pi \hbar} \int \int d\lambda \, dv' e^{im_*(v-v')\lambda} f(x, v', t) \\
\times \left[ V \left( x + \frac{\lambda \hbar}{2}, t \right) - V \left( x - \frac{\lambda \hbar}{2}, t \right) \right] = 0
\]

\( f(x, v, t) \) = Wigner pseudo – probability distribution

- Total potential: \( V = V_H + V_{\text{conf}} \)

- Poisson’s equation for the Hartree potential:

\[
\frac{\partial^2 V_H}{\partial x^2} = \frac{e^2}{\varepsilon} \int_{-\infty}^{\infty} f \, dv
\]

- Anharmonic confinement:

\( V_{\text{conf}}(x) = \frac{1}{2} m_* \omega_0^2 (x^2 + Kx^4) \), \( K \ll 1 \)
**Dimensionless parameters**

- **Normalized Planck constant**
  \[ H = \frac{\hbar \omega_0}{k_B T_e} = \frac{\hbar}{\sigma_x \sigma_p} \]

- **Ratio of electron plasma frequency to confinement frequency**
  \[ \eta = \frac{\omega_p^2}{\omega_0^2} \]
  \[ \omega_p = (e^2 n_e / m \varepsilon_0)^{1/2} \]
Time-evolution of the Wigner-Poisson system

Initial condition:

\[ f_0(x, v) = \frac{n_e}{\sqrt{2\pi} \sigma_p} \exp\left(-\frac{(x - x_0)^2}{2\sigma_x^2} - \frac{m^* v^2}{2\sigma_p^2}\right) \]

- We solve the Wigner Poisson equations for **two almost identical initial conditions**, which differ for a small initial perturbation.
- **Perturbation**: Small kick either in real space \((x \rightarrow x + \delta x_0)\) or in velocity space \((v \rightarrow v + \delta v_0)\)
- Then compute the quantum fidelity:

\[ F(t) = \frac{2\pi \hbar}{m^* N^2} \iint f_1(x, v, t) f_2(x, v, t) \, dx \, dv \]
Quantum fidelity — results

\[ F(t) = \frac{2\pi \hbar}{m* N^2} \int \int f_1(x, v, t) f_2(x, v, t) \, dx \, dv \]

\( \tau_C = \text{critical time} \)

\[ \tau_C = -\tau_0 \ln \delta v_0 + \text{const.} \]

Initial perturbation
Trajectory separation

\[ \langle v_i(t) \rangle = \int \int f_i(x, v, t) v \, dx \, dv, \; i = 1, 2 \]

\[ \delta v(t) = \langle v_1(t) \rangle - \langle v_2(t) \rangle \]

Obviously \( \delta v(t = 0) = \delta v_0 \).

\[ \eta = 6 \]

\[ \eta = 8 \]

\[ \lambda = 0.9 T_0^{-1} \]

\[ \lambda = 1.2 T_0^{-1} \]

\[ \delta v(t) = \delta v_0 \exp(\lambda t) \]

\( \lambda \) is a sort of “Lyapunov exponent”
Trajectory separation and critical time

The fidelity drop occurs when the trajectory separation reaches a critical value \( \delta v_C \simeq \hbar / m_* \sigma_x = H v_{th} \).

This means that the initial perturbation has been amplified up to a magnitude comparable with Planck’s constant.

Substituting the expression for \( \delta v_C \) in \( \delta v(t) = \delta v_0 \exp(\lambda t) \), we obtain:

\[
\tau_C = -\frac{1}{\lambda} \left[ \ln \left( \frac{\delta v_0}{v_{th}} \right) - \ln H \right]
\]

which correctly reproduces the numerical result (both the slope and the constant): \( \tau_C = -\tau_0 \ln \delta v_0 + \text{const.} \).
Summary of key results

- **Critical time is linked to trajectory separation** ("Lyapunov exponent")

- **Sudden drop (instead of exponential decay) is a nonlinear effect**
  - When $f_1$ and $f_2$ start to diverge, also the Hamiltonian diverges (nonlinearity)
  - The Hamiltonian, in turns, acts on the evolution of $f_1$ and $f_2$, and so on.
  - The outcome is a faster-than-exponential decay.
Quantum fidelity for other N-body system

System of Interacting Electrons
Self-consistent set of quantum hydrodynamic equations
Periodic boundary conditions

Trapped Bose-Einstein Condensate
Gross-Pitaevskii equation (nonlinear Schrödinger eq.)

Quantum Wells
Self-consistent Wigner–Poisson system
Conclusions

• **Stability of many-electron systems**
  – Unusual behavior of the quantum fidelity.
  – Verified for 3 different types of modeling (all mean-field type)
    ➢ Quantum hydrodynamics
    ➢ Gross-Pitaevskii equation (NLSE)
    ➢ Wigner-Poisson model
  – Is it typical of N-body interacting systems?

• **Perspectives**
  – Exact N-body problem
  – Under way: N=2 interacting electrons in a nonparabolic confinement
Effect of environmental decoherence

\[ \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{im_x}{2\pi\hbar} \int \int d\lambda d\nu e^{im_x(\nu-\nu')} \lambda f(x', \nu', t) \times \left[ V \left( x + \frac{\lambda\hbar}{2}, t \right) - V \left( x - \frac{\lambda\hbar}{2}, t \right) \right] = \left( \frac{\partial f}{\partial t} \right)_{\text{scatt}} \]

\[ \left( \frac{\partial f}{\partial t} \right)_{\text{scatt}} = 2\gamma \frac{\partial (vf)}{\partial \nu} + D_v \frac{\partial^2 f}{\partial \nu^2} + D_x \frac{\partial^2 f}{\partial x^2}, \quad D_v D_x \geq \gamma^2 \hbar^2 / 4m_x^2 \]

\[ D_v = \gamma v_{\text{th}}^2 \]

Quantum fidelity

\[ F(t) \]

\[ \gamma = 0 \]

\[ \gamma = 5 \times 10^{-4} \]